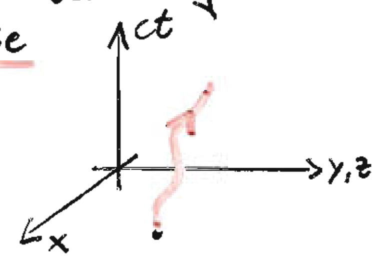
Die Dynamik des rel. Strings

1 Wdh. - Das rel. Punktteilchen

Ereignisse werden im Minkowskiraum beschr.: $x^\mu = (ct, x, y, z)$
Raum-Zeit $\mathbb{R}^{1,3}$

Im Laufe der Zeit bewegt sich ein rel. Pkt.teilchen in diesem $\mathbb{R}^{1,3}$
=> Weltlinie



Parameterdarstllg.
$\gamma : D \subseteq \mathbb{R} \to W \subseteq \mathbb{R}^4$

Lorentz- u. parameterisierungsinvar. metr. Tensor

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

1.1 Prinzip d. Wirkung

$$\boxed{S = -mc \int_\gamma ds} \qquad S[x^\mu + \delta x^\mu] = S[x^\mu]$$

$$S = -mc \int \sqrt{\dot{x}^{\mu 2}}\, dt \quad \Big| \; \dot{x}^{\mu 2} = c^2 - v^2$$
$$= -mc^2 \int \sqrt{1 - \frac{v^2}{c^2}}\, dt$$

Variation: $\delta S = 0 = -mc \int \delta(ds)$

$$\boxed{\frac{dp^\mu}{d\tau} = \frac{d\left(mc \frac{dx^\mu}{ds}\right)}{d\tau} = 0}$$

1.2 Übergang zur klass. Mechanik

$$S = -mc^2 \int \sqrt{1 - \frac{v^2}{c^2}}\, d\tau \qquad NR: \underset{|x|<1}{(1+x)^{1/2}} \approx 1 + \frac{x}{2}$$

$\downarrow v \ll c$

$$\approx -mc^2 \int d\tau + \underbrace{mc^2 \int \frac{v^2}{2c^2} d\tau}$$

Klass. Lagrangeformalismus: $S = \int_{t_A}^{t_E} \frac{m}{2} v^2 + V(x(t))\, dt$

2 Der rel. String

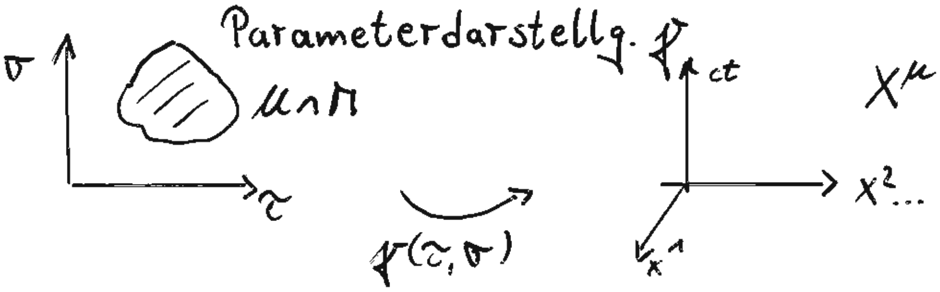
String: elementarsten Objekte
↳ elementare Schwingungsmoden: Teilchen
↳ Beschreiben v. Weltblättern

Raum-Zeit-Koord.: $X^\mu = (X^0, \dots, X^d) = (ct, x, y, z, \dots)$

Metr. Tensor: $\eta_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \\ & & & & \ddots \end{pmatrix}$

Minkowskimetrik: $ds^2 = c^2 dt^2 - \sum_{i=1}^{d} dx^{i2}$

2.1 Verlauf eines Strings



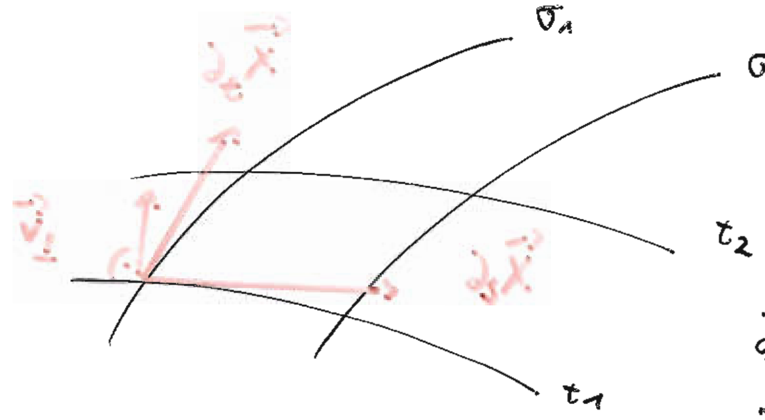
2.2 Nambu-Goto-Wirkung

Fläche in $\mathbb{R}^3$: $A = \int \sqrt{\partial_a f^2 \partial_b f^2 - 2 \partial_a f \partial_b f}\, \mu(a,b)$; $f(a,b)$

Stringwirkg.: $S = -\frac{T_0}{c} \int_{\tau_A}^{\tau_E} d\tau \int_0^{\sigma_1} d\sigma \sqrt{2\dot{X}X' - \dot{X}^2 X'^2}$; $X^\mu(\tau,\sigma)$

$\dot{X} = \frac{\partial X^\mu}{\partial \tau}$ $\quad X' = \frac{\partial X^\mu}{\partial \sigma}$

Variation: $S[X^\mu] = S[X^\mu + \delta X^\mu]$



$$\partial_\sigma \vec{X}(\tau,\sigma) = \frac{\partial \vec{X}}{\partial s}\frac{ds}{d\sigma}$$

$$\partial_s \vec{X}^2 = 1$$

$$\vec{v}_\perp = \partial_t \vec{X} - (\partial_t \vec{X} \cdot \partial_s \vec{X})\,\partial_s \vec{X}$$

2.5 String mit offenen Enden

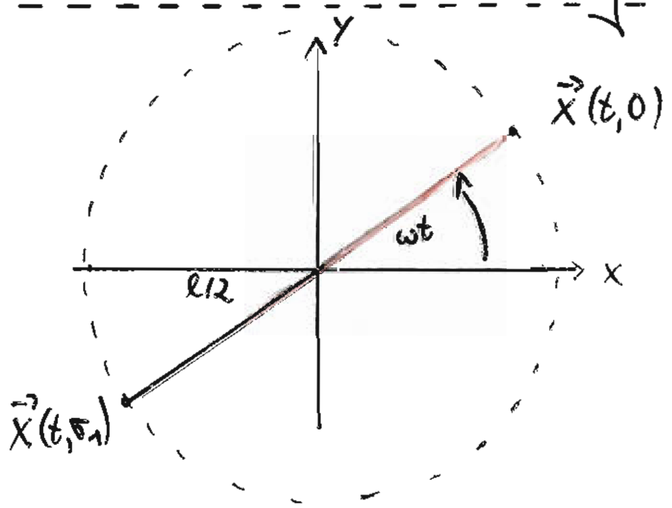
$P^\sigma_\mu(\tau, \sigma_{A,E}) = 0$ für alle μ $\Rightarrow$ c u. Δ an den Stringenden

2.6 Schwingungsmoden des Strings

$\partial_\sigma \vec{X} \cdot \partial_t \vec{X} = 0 \Rightarrow \vec{v}_\perp = \partial_t \vec{X}$ an allen Pkt. $\Rightarrow \vec{v} := \partial_t \vec{X} = \vec{v}_\perp$

- Wellengl.: $\partial_\sigma^2 \vec{X} - \frac{1}{c^2}\partial_t^2 \vec{X} = 0$ $\quad (\partial_\sigma \vec{X})^2 + \frac{1}{c^2}(\partial_t \vec{X})^2 = 1$
- $\partial_t \vec{X} \cdot \partial_\sigma \vec{X} = 0$ $\quad \partial_\sigma \vec{X}\big|_{\sigma=0,\sigma_1} = 0$

2.7 Der starre „2-dim." String



$$\vec{X}(t,\sigma) = \frac{\sigma_1}{\pi}\cos\frac{\pi\sigma}{\sigma_1}\left(\cos\frac{\pi c t}{\sigma_1}, \sin\frac{\pi c t}{\sigma_1}\right)$$

2.3 Bewegungsgleichung

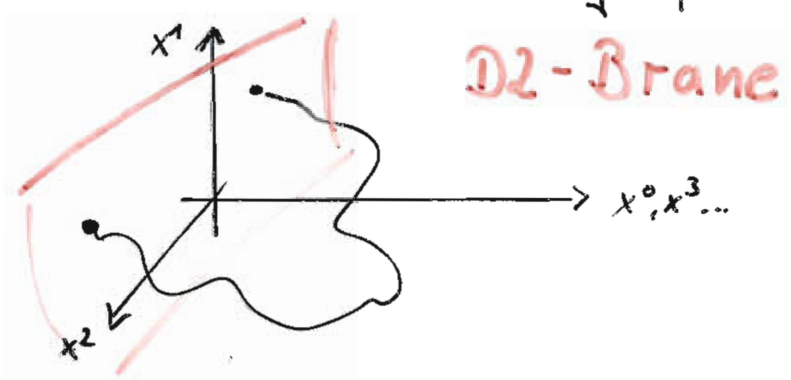
$$\delta S = 0 \Rightarrow \boxed{\frac{\partial P^\tau_\mu}{\partial \tau} + \frac{\partial P^\sigma_\mu}{\partial \sigma} = 0}$$

- Randbedingung: $\partial_\tau X^0(\tau, \sigma_{A,E}) \neq 0 \quad P^\sigma_0(\tau, \sigma_{A,E}) = 0$

  Dirichlet: $\partial_\tau X^\mu(\tau, \sigma_{A,E}) = 0 \Rightarrow \delta X^\mu(\tau, \sigma_{A,E}) = 0$

  Neumann: $P^\sigma_\mu(\tau, \sigma_{A,E}) = 0$ offene Enden

- D_p-Brane: p-dim. Objekt, auf welchem Dirichlet hervorgerufen



2.4 Statische Eichung

$$t(\tau, \sigma) = \tau \Rightarrow X^\mu = (c\tau, x^1, \dots) = (c\tau, \vec{X}(\tau, \sigma))$$